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Three-fluid heat exchangers with three thermal communications. Part A: general mathematical model

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Abstract

A general, simple and easy to implement analytical model that enables design and analysis of three-fluid heat exchangers with three thermal communications for all flow arrangements is developed. The model is shown to reduce to either a three-fluid, two thermal communications model or the traditional two-fluid heat exchanger model under proper conditions. Six non-dimensional design parameters are identified and their effect on the temperature distributions of the different fluid streams is presented. The model shows that the presence of three coupled thermal interactions makes temperature distributions of the different streams difficult to assess due to slight changes in the design parameters.

1. Introduction

Three-fluid heat exchangers are used in a variety of applications. Some of the popular uses of these exchangers are found in the petro-chemical, aerospace and chemical industries. Air separation systems, helium–air separation units, systems that deal with purification and liquefaction of hydrogen, and ammonia gas synthesis typically use three-fluid heat exchangers [1]. A variety of microscale heat exchangers with two internal working fluids where thermal insulation is limited by size constraints and all typical two-fluid heat exchangers can also be treated as three-fluid heat exchangers where the third fluid is the ambient with infinite thermal capacity.

Several researchers have presented a general analytical procedure to obtain the temperature distribution in all of the fluid streams in multi-stream, one-dimensional heat exchangers assuming that there were no multiple eigenvalues to the solution [2,3]. However, no explicit solution to the problem of three-fluid heat exchangers with three thermal interactions has been derived for any/all fluid flow arrangements. Several others have presented explicit/iterative flow direction dependent solutions for this class of three-fluid heat exchangers [4–7] assuming that multiple eigenvalues do not exist. The only three-fluid models which are valid for three thermal communications and also for multiple zero eigenvalues were developed by Aulds [8] and Aulds and Barron [9]. However, they only considered the case in which fluids 1 and 3 flow in parallel, with the second fluid flowing in the counter direction (our case P2, as shown in Fig. 1). An extensive review of the work related to three-fluid heat exchangers has been provided by Sekulic and Shah [1].

Ameel and Hewavitharana [10] have developed a model of a two-fluid counter current heat exchanger where both fluids are subjected to external heating. Ameel [11] has developed another model of a two-fluid parallel flow heat exchanger where again both fluids can interact with the ambient. Both models [10,11] have been developed assuming that multiple eigenvalues do not exist. As the ambient can be considered a third fluid with infinite thermal capacity, both models [10,11] can be considered special cases of three-fluid heat exchangers with three thermal communications. Barron [12] has also developed a model where one of the fluids in a two-fluid

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Nomenclature

a_{ij}	constants, $i = 1,2,3$, $j = 1,2,3$
s_j	constants, $j = 1,2,3$
t_j	constants, $j = 1,2,3$
u_j	constants, $j = 1,2,3$
v_j	constants, $j = 1,2,3$
w_j	constants, $j = 1,2,3$
Y_j	constants, $j = 1,2,3$
Z_j	constants, $j = 1,2,3$
C_{12}	ratio of the thermal capacity of fluid 1 to 2, $(\dot{m}c_p)_1/(\dot{m}c_p)_2$
C_{32}	ratio of the thermal capacity of fluid 3 to 2, $(\dot{m}c_p)_3/(\dot{m}c_p)_2$
D_j	eigenvalues, $j = 1,2,3$
e_i	$(\theta_{3,\xi=1} - \theta_{3,in})/(1 - \theta_{3,in})$
e_c	$\theta_{1,\xi=1}$
NTU_1	number of transfer units based on the heat exchange between fluids 1 and 2 and thermal capacity of fluid 1, $(UA)_{12}/(\dot{m}c_p)_1$
R_1	ratio of the thermal resistances between fluids 1 and 2 and fluids 3 and 2, $(UA)_{32}/(UA)_{12}$
R_2	ratio of the thermal resistances between fluids 1 and 2 and fluids 3 and 1, $(UA)_{31}/(UA)_{12}$

Greek symbols

ξ	non-dimensional length, x/L
θ_i	non-dimensional temperature, $(T_i - T_{1,in})/(T_{2,in} - T_{1,in})$
$\theta_{3,in}$	non-dimensional inlet temperature of the third fluid, $(T_{3,in} - T_{1,in})/(T_{2,in} - T_{1,in})$

Subscripts

i	fluid stream $i = 1,2,3$
in	position where fluid enters the heat exchanger
out	position where fluid leaves the heat exchanger

heat exchanger is interacting with the ambient. Figs. 1 and 2, respectively, show schematics of possible tube arrangements and all possible flow arrangements for three-fluid heat exchangers with three thermal communications.

It can be realized from the literature that a unified theory where a single solution can be used to obtain the temperature distribution in all of the streams for all fluid flow arrangements and which is valid for multiple zero as well as non-zero eigenvalues is lacking for three-fluid heat exchangers with three thermal communications. Also, no parametric study of the complex behavior of all of the design parameters on the temperature distribution of different streams has ever been presented/analyzed for this class of heat exchangers. A unified, flow direction independent, non-dimensional model for three-fluid heat exchangers with two thermal communications has been developed for all possible fluid flow cases by Sekulic and Shah [1]. The need for a general model for a three-fluid heat exchanger with three thermal communications is expressed in their paper. In their own words "A similar theoretical/analytical approach should be extended to other classes of three-fluid heat exchangers, for example three-fluid heat exchangers with all three fluids in thermal contact". This analysis fills many existing gaps in this specialized heat exchanger design problem and will facilitate improved design for three-fluid heat exchangers. It is also mentioned by Sekulic and Shah [1] that further studies should be conducted on the overall performance of the three-fluid heat exchanger as well as on reconsideration of the overall three-fluid heat exchanger effectiveness. This concern is addressed in a second paper [13].

In the present study, six non-dimensional design parameters are explicitly identified and studied to develop insights into the behavior of this class of heat exchangers. The design parameters include the ratio of the thermal resistances between fluid streams 1 and 2 and fluid streams 3 and 2 (R_1), the ratio of the thermal resistances between fluid streams 1 and 2 and fluid streams 3 and 1 (R_2), the capacitance ratio for fluids 1 and 2 (C_{12}), the capacitance ratio for fluids 3 and 2 (C_{32}), the number of transfer units based on fluid 1 (NTU_1), and the dimensionless inlet temperature for fluid 3 ($\theta_{3,in}$). Resistance $(1/(UA))$ between any two fluid streams is defined as the inverse of the product of the over all heat transfer coefficient (U) and contact area (A) through which heat is exchanged. Capacity (C) of any fluid stream is defined as the product of mass flow rate ($\dot{m}$) and its specific heat (c_p). Six different non-dimensional effectiveness parameters or figures of merit are also identified based on the engineering goals of the heat exchangers. These goals include heating a cold fluid, cooling a hot fluid, heating or cooling an intermediate temperature fluid stream or maximizing the heat transfer from the middle fluid stream to other two streams. Various issues related to the evaluation of the effectiveness of three-fluid heat exchangers are discussed in a subsequent paper [13].

2. Mathematical model

In a three-fluid single-pass heat exchanger with three thermal communications three different fluid streams

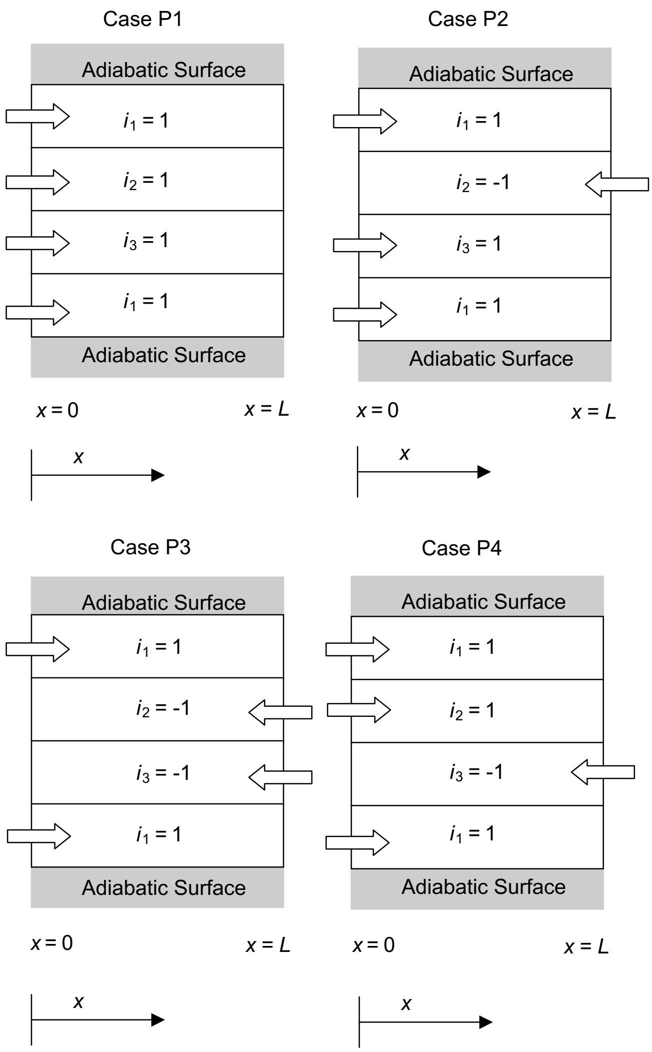

Fig. 1. Possible flow arrangements for three-fluid heat exchangers with three thermal communications.

interact with one another to reach their respective thermal equilibria. Depending on the flow directions, four different flow arrangements are possible (Fig. 1). Sekulic and Shah [1] have presented and named all four flow arrangements for three-fluid heat exchangers with two thermal communications. The same nomenclature is used here in order to make this study compatible with their work.

The symbol i_j ($j = 1, 2, 3$) is used to represent the flow direction of the three streams. Positive i_j means that fluid is flowing in the positive x-direction; negative i_j represents flow of fluid in the negative x-direction (Fig. 1). The relative temperature of the three fluids is neither specified nor used in the analysis. Therefore, the model is also general in the sense that any of the three-fluid streams can represent the hot, cold, or intermediate temperature fluids.

In order to solve for the axial temperature distributions of all fluids in a three-fluid heat exchanger with three thermal communications, a number of idealizations and assumptions are made, including:

(1) The heat exchanger operates at a steady state.
(2) The mass flow rates are constant.
(3) All properties, variables, and parameters, e.g., C_{12}, C_{32}, NTU_1, R_1, R_2 are constant.
(4) No heat loss occurs to the ambient.
(5) No axial conduction occurs in the pipes or in the fluid streams.
(6) The temperature distribution within a stream in the transverse direction is assumed to be uniform and equal to the average temperature of the fluid stream.
(7) No heat source or heat sink exists in the heat exchanger or in any of the fluid streams.

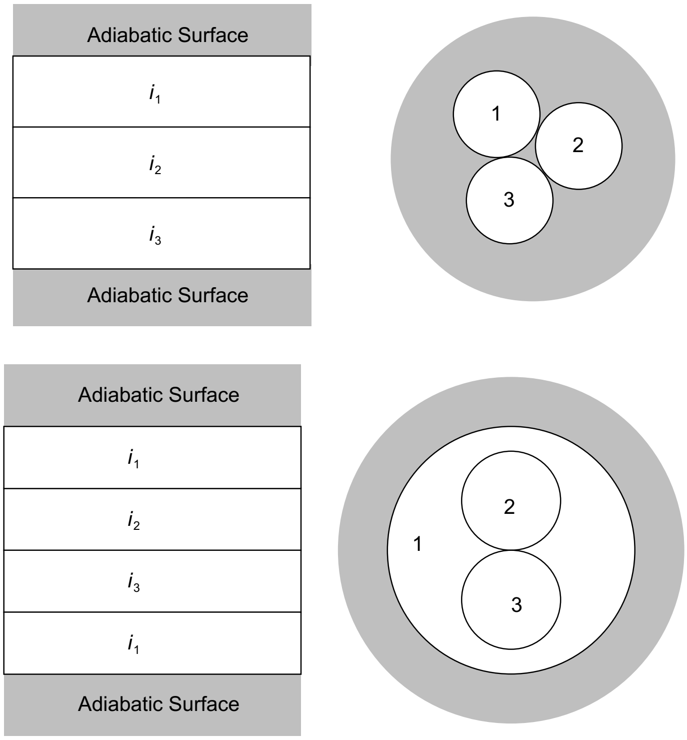

Fig. 2. Schematic and possible pipe arrangement for a three-fluid heat exchanger with three thermal communications.

(8) Each fluid stream is exchanging heat with the other two fluid streams.
(9) Either there is no phase change or phase change occurs at a constant temperature in the fluid streams.
(10) The heat transfer area is uniformly distributed along the heat exchanger.

Consider case P1 in which all three fluids flow concurrently. With the idealizations and assumptions stated previously, an energy balance on a differential length dx for fluid 1 (Fig. 3) gives,

$$\left[\dot{m}c_pT - \dot{m}c_p\left(T + \frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x\right)\right]_1 + \frac{(\mathrm{UA})_{21}}{L}(T_2 - T_1)\,\mathrm{d}x + \frac{(\mathrm{UA})_{31}}{L}(T_3 - T_1)\,\mathrm{d}x = 0 \tag{1}$$

If fluid F1 is flowing in the negative x-direction, the energy balance on the same differential length dx gives,

$$-\left[\dot{m}c_pT - \dot{m}c_p\left(T + \frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x\right)\right]_1 + \frac{(\mathrm{UA})_{21}}{L}(T_2 - T_1)\mathrm{d}x + \frac{(\mathrm{UA})_{31}}{L}(T_3 - T_1)\,\mathrm{d}x = 0 \tag{2}$$

Eqs. (1) and (2) can be combined into a single equation by using variable i_1 which takes a value of +1 for the flow of fluid 1 (F1) in the positive x-direction and −1 if F1 is flowing in the negative x-direction.

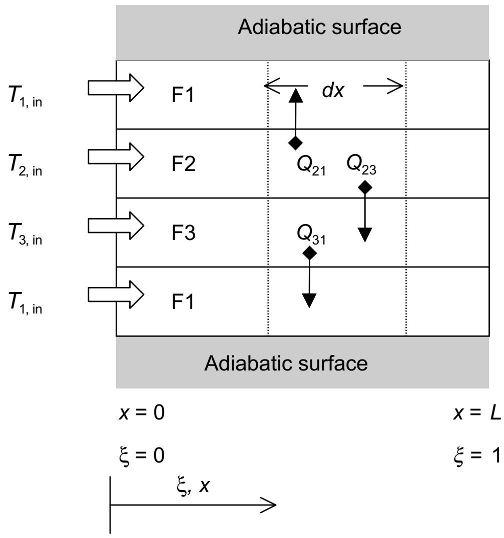

Fig. 3. Schematic diagram for case P1.

$$i_1\left[\dot{m}c_pT - \dot{m}c_p\left(T + \frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x\right)\right]_1 + \frac{(\mathrm{UA})_{21}}{L}(T_2 - T_1)\,\mathrm{d}x + \frac{(\mathrm{UA})_{31}}{L}(T_3 - T_1)\,\mathrm{d}x = 0 \tag{3}$$

Similarly, differential energy balance equations for fluids 2 (F2) and 3 (F3) can be written as,

$$i_2\left[\dot{m}c_pT - \dot{m}c_p\left(T + \frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x\right)\right]_2 + \frac{(\mathrm{UA})_{21}}{L}(T_1 - T_2)\,\mathrm{d}x + \frac{(\mathrm{UA})_{32}}{L}(T_3 - T_2)\,\mathrm{d}x = 0 \quad (4)$$

$$i_3\left[\dot{m}c_pT - \dot{m}c_p\left(T + \frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x\right)\right]_3 + \frac{(\mathrm{UA})_{32}}{L}(T_2 - T_3)\,\mathrm{d}x + \frac{(\mathrm{UA})_{31}}{L}(T_1 - T_3)\,\mathrm{d}x = 0 \quad (5)$$

On simplifying, Eqs. (3)–(5) reduce to,

$$i_1\frac{\mathrm{d}T_1}{\mathrm{d}x} + \frac{(\mathrm{UP})_{21}}{(\dot{m}c_p)_1}(T_1 - T_2) + \frac{(\mathrm{UP})_{31}}{(\dot{m}c_p)_1}(T_1 - T_3) = 0 \quad (6)$$

$$i_2\frac{\mathrm{d}T_2}{\mathrm{d}x} + \frac{(\mathrm{UP})_{21}}{(\dot{m}c_p)_2}(T_2 - T_1) + \frac{(\mathrm{UP})_{23}}{(\dot{m}c_p)_2}(T_2 - T_3) = 0 \quad (7)$$

$$i_3\frac{\mathrm{d}T_3}{\mathrm{d}x} + \frac{(\mathrm{UP})_{23}}{(\dot{m}c_p)_3}(T_3 - T_2) + \frac{(\mathrm{UP})_{31}}{(\dot{m}c_p)_3}(T_3 - T_1) = 0 \quad (8)$$

Eqs. (6)–(8) are valid for all four flow arrangements given in Fig. 1. Using non-dimensional parameters and with some rearrangements, Eqs. (6)–(8) can be rewritten as,

$$i_1\frac{\mathrm{d}\theta_1}{\mathrm{d}\xi} = \mathrm{NTU}_1[(\theta_2 - \theta_1) + R_2(\theta_3 - \theta_1)] \quad (9)$$

$$i_2\frac{\mathrm{d}\theta_2}{\mathrm{d}\xi} = \mathrm{NTU}_1C_{12}[(\theta_1 - \theta_2) + R_1(\theta_3 - \theta_2)] \quad (10)$$

$$i_3\frac{\mathrm{d}\theta_3}{\mathrm{d}\xi} = \mathrm{NTU}_1\frac{C_{12}}{C_{32}}[R_1(\theta_2 - \theta_3) + R_2(\theta_1 - \theta_3)] \quad (11)$$

Eqs. (9)–(11) are micro energy balance equations. They satisfy the energy balance at every cross-section along the length of the heat exchanger and represent three equations for the unknown temperatures θ_1, θ_2 and θ_3. Since there is no heat sink or source available inside the heat exchanger, the summation of all energy given or received at any cross-section in the heat exchanger, and thus throughout the heat exchanger, must be zero. This suggests that Eqs. (9)–(11) are not independent equations. Therefore, an another micro energy balance heat equation can be formulated by adding all the inter-fluid heat transfer within the heat exchanger. The direction of fluid streams is taken care of by again introducing i_1, i_2, and i_3. For an infinitesimal element of length dx (Fig. 3) along the axial direction of the heat exchanger, the micro energy balance equation in non-dimensional form is,

$$i_1C_{12}\frac{\mathrm{d}\theta_1}{\mathrm{d}x} + i_2\frac{\mathrm{d}\theta_2}{\mathrm{d}x} + i_3C_{32}\frac{\mathrm{d}\theta_3}{\mathrm{d}x} = 0 \quad (12)$$

Together, Eqs. (9), (10) and (12) along with the non-dimensional boundary conditions given in Table 1 for all four fluid flow arrangements represent a complete eigenvalue problem. This eigenvalue problem can be solved to obtain three eigenvalues D_1, D_2, and D_3 which will determine the temperature distribution of all three fluids for all four cases. Assuming that these three eigenvalues D_1, D_2, and D_3 are different from each other, the final solution in an abridged form can be presented as follows:

$$\theta_i = \sum_{j=1}^{3} a_{ij}\exp(D_j\xi) \quad i = 1,2,3 \quad (13)$$

where,

$$D_1 = 0 \quad (14)$$

$$D_2 = (-B + (B^2 - 4AC)^{1/2})/(2C) \quad (15)$$

$$D_3 = (-B - (B^2 - 4AC)^{1/2})/(2C) \quad (16)$$

with,

$$A = (R_1 + R_2 + R_1R_2)(-i_1C_{12} - i_2 - i_3C_{32})$$

$$B = (-i_1i_2(R_1 + R_2) - i_1i_3C_{32}(1 + R_1) - i_3i_2C_{32}(1 + R_2)/C_{12})/\mathrm{NTU}_1$$

$$C = -i_1i_2i_3C_{32}/(\mathrm{NTU}_1^2C_{12})$$

The constants a_{ij} can be determined as follows:

$$s_j = a_{1j}/a_{2j} \quad j = 1,2,3 \quad (17)$$

$$u_j = a_{3j}/a_{2j} \quad j = 1,2,3 \quad (18)$$

with,

$$s_j = \frac{R_1 + R_2 + R_1R_2 + \frac{i_2D_jR_2}{\mathrm{NTU}_1C_{12}}}{R_1 + R_2 + R_1R_2 + \frac{i_1D_jR_1}{\mathrm{NTU}_1}} \quad j = 1,2,3 \quad (19)$$

Table 1
Boundary conditions for all four cases in non-dimensional form

	P1		P2		P3		P4	
j	ξ	θ_j	ξ	θ_j	ξ	θ_j	ξ	θ_j
1	0	0	0	0	0	0	0	0
2	0	1	1	1	1	1	0	1
3	0	$\theta_{3,\mathrm{in}}$	0	$\theta_{3,\mathrm{in}}$	1	$\theta_{3,\mathrm{in}}$	1	$\theta_{3,\mathrm{in}}$

$$u_j = \frac{\left[\frac{i_1 D_j}{\mathrm{NTU}_1} + 1 + R_2\right] s_j - 1}{R_2} \quad \text{or}$$

$$u_j = \frac{\left[\frac{i_2 D_j}{\mathrm{NTU}_1 C_{12}} + 1 + R_1\right] - s_j}{R_1} \qquad j = 1, 2, 3 \tag{20}$$

and,

$$a_{21} = \frac{s_3((w_2 s_3 - s_2 w_3) - \theta_{3,\mathrm{in}}(v_2 s_3 - s_2 v_3))}{((v_1 s_3 - s_1 v_3)(w_2 s_3 - s_2 w_3) - (w_1 s_3 - s_1 w_3)(v_2 s_3 - s_2 v_3))} \tag{21}$$

$$a_{22} = \frac{s_3 - (v_1 s_3 - s_1 v_3) a_{21}}{(v_2 s_3 - s_2 v_3)} \tag{22}$$

$$a_{23} = \frac{-(s_1 a_{21} + s_2 a_{22})}{s_3} \tag{23}$$

where, for $(j = 1, 2, 3)$ cases

P1 and P2, $\quad w_j = u_j \qquad (24)$

P3 and P4, $\quad w_j = u_j \exp(D_j) \qquad (25)$

P1 and P4, $\quad v_j = 1$ and $\qquad (26)$

P2 and P3, $\quad v_j = \exp(D_j) \qquad (27)$

In implementing this model, either form of Eq. (20) may be used. Selection may be based on reducing undesirable effects produced by either R_1 or R_2 approaching to values of zero or infinity. The solution presented above, like all other existing models, is not valid when multiple eigenvalues are present. An additional solution, valid when multiple eigenvalues exist, is presented in Appendix A.

3. Results and discussion

The model developed here considers all possible thermal interactions and flow arrangements and is therefore the most general model developed for three-fluid, single pass, parallel flow heat exchangers. The new model is verified by comparison to five previously reported models for heat exchangers operating under less severe constraints than the three-fluid three thermal communication arrangement. The five models selected for comparisons are:

1. The standard two-fluid single pass parallel and counter flow heat exchanger model with no thermal interaction with the atmosphere.
2. A two-fluid, single pass, parallel, co-current flow heat exchanger model with ambient thermal interaction [11].
3. A two-fluid, single pass, parallel, counter-current flow heat exchanger model with ambient thermal interaction [10].
4. A three-fluid, two thermal communications heat exchanger model [1].
5. A three-fluid, three thermal communications model for the P2 arrangement [9].

The conditions required for the new model to simulate each of the five test models are given in Table 2. The three-fluid, three thermal communication general model with appropriate conditions selected (Table 2), is found to produce the same temperature distribution for the fluid streams for each of the five test models [14]. Details of these comparisons can be found in [14]. As an example, we present in Table 3 a comparison between our general model and the three-fluid three thermal communication model developed by Aulds and Barrron [9] (Aulds and Barron [9] developed solution for case P2 only). As another example, a comparison is presented in Table 4 between the present solution and the P1 case of three-fluid two thermal communication model developed by Sekulic and Shah [1].

Next, the solution shows that the temperature distributions of the streams are functions of six design parameters (i.e., R_1, R_2, C_{12}, C_{32}, NTU_1, and $\theta_{3,\mathrm{in}}$). Therefore, it is necessary to consider the effect of these six design parameters on the temperature distributions of the different streams and different effectivenesses of the heat exchangers.

As shown in Fig. 1, there are four possible flow arrangements for three-fluid heat exchangers with three thermal communications. Cases P2, P3 and P4 are, however, not altogether different from each other. Each of these arrangements has two streams flowing in one direction and one in the opposite direction. Although, the flow directions in these cases for the different streams are not the same (Fig. 1), they are related to each other because all of the streams interact with one another and therefore affect the temperature distributions of the

Table 2
Limiting conditions necessary to reduce the general model to given models

Heat exchanger models	Conditions
Two-fluid with one thermal communication	$C_{32} = \infty$, $R_1 = R_2 = 0$
Two-fluid with three thermal communications [11]	$C_{32} = \infty$
Two-fluid with three thermal communications [10]	$C_{32} = \infty$
Three-fluid with two thermal communications [1]	$R_2 = 0$
Three-fluid with three thermal communications [9]	Same as case P2

Table 3
A comparison between the intermediate and cold fluid temperature effectiveness values for selected combinations of the control parameters predicted by (a) Aulds and Barron [9] and (b) this work [14]

$X = 1 - \theta_{3,in}$	NTU	R_1/R_2	R_1/R_3	C_c/C_h	C_i/C_h	e_i		e_c	
						a	b	a	b
0.756	0.197	3.68	1.066	0.383	0.682	0.0445	0.0445	0.2436	0.2436
0.281	0.176	3.08	0.743	0.938	1.579	−0.4252	−0.4252	0.3639	0.3639
0.068	0.121	4.14	1.106	1.723	1.89	−3.6038	−3.6038	0.3699	0.3699
0.486	0.151	1.16	0.757	0.953	0.76	−0.0539	−0.0539	0.1925	0.1925
0.471	0.136	0.487	0.707	1.238	1.061	0.0143	0.0143	0.1436	0.1436

Table 4
A comparison between the non-dimensional outlet temperatures for various NTU_1 values for case P1 predicted by (a) Sekulic and Shah [1] and (b) this work

NTU_1	$\theta_{1,out}$		$\theta_{2,out}$		$\theta_{3,out}$	
	a	b	a	b	a	b
0.0	0.0000	0.0000	1.0000	1.0000	0.5000	0.5000
1.0	0.3110	0.3110	0.3774	0.3774	0.5007	0.5007
2.0	0.3458	0.3458	0.3646	0.3646	0.4298	0.4298
3.0	0.3584	0.3584	0.3665	0.3665	0.3960	0.3960
4.0	0.3639	0.3639	0.3675	0.3675	0.3807	0.3807
5.0	0.3664	0.3664	0.3680	0.3680	0.3739	0.3739

The values chosen for different design parameters are: $R_1 = 0.3$, $R_2 = 0$, $C_{12} = 2.0$, $C_{32} = 0.8$, $\theta_{3,in} = 0.5$.

streams in contact along the length of the heat exchanger. It can be shown that fluid streams 1, 2 and 3 of case P2 will be thermally identical to fluid streams 1, 3 and 2 of case P4, respectively, if proper values of design parameters are chosen [14].

Similarly, it can also be shown that streams 1, 2 and 3 of case P3 will be thermally identical to streams 3, 2 and 1 of case P4, respectively. Note that for case P3 the parallel streams start from $\xi = 1$, whereas for case P4, the parallel streams start from $\xi = 0$. Therefore, the direction of the flow of the different streams must be taken into account in making the comparisons. Thus, temperatures at location $\xi = \xi_1$ for case P4 are related to the temperatures at location $\xi = (1 - \xi_1)$ of corresponding streams for case P3 [14].

This discussion indicates that there are only two major cases, P1 and P2, that need to be studied to understand the behavior of these types of heat exchangers. Also, it has been found that the effect of different parameters on cases P1 and P2 are similar. As case P2 is more widely used and more effective, the effect of all six design parameters on the temperature distributions of the three streams is presented here for case P2 only. To make the interpretation of the effect of different design parameters easier to ascertain, it is assumed that fluids 1, 2 and 3 represent cold, hot, and intermediate fluid temperature streams, respectively. However, the results and conclusions derived are general and can be applied to any combination of the three fluid streams with different temperatures.

The effect of R_1 on the temperature distributions of all the three streams for case P2 is shown in Fig. 4. R_1 is defined as the ratio of the overall heat transfer resistance between fluid streams 1 and 2 to that between fluid streams 3 and 2. Therefore, as the value of R_1 increases, the overall heat transfer resistance between fluid streams 3 and 2 decreases relative to the other resistance. This leads to the relatively higher thermal interaction between

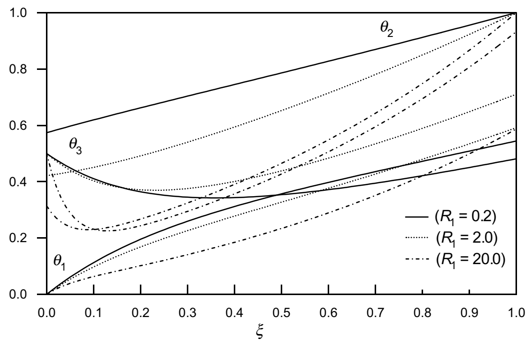

Fig. 4. Effect of R_1 on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, $NTU_1 = 1.0$, and $\theta_{3,in} = 0.5$.

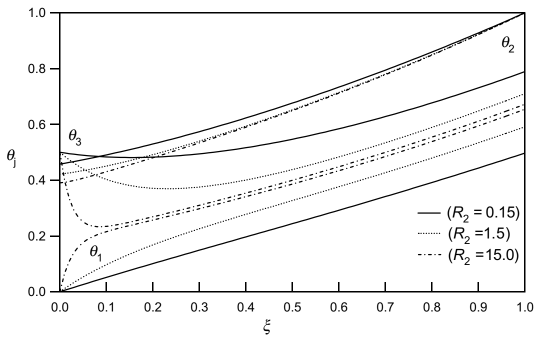

Fig. 5. Effect of R_2 on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_1 = 2.0$, $C_{12} = 0.5$, $C_{32} = 0.8$, $\mathrm{NTU}_1 = 1.0$, and $\theta_{3,\mathrm{in}} = 0.5$.

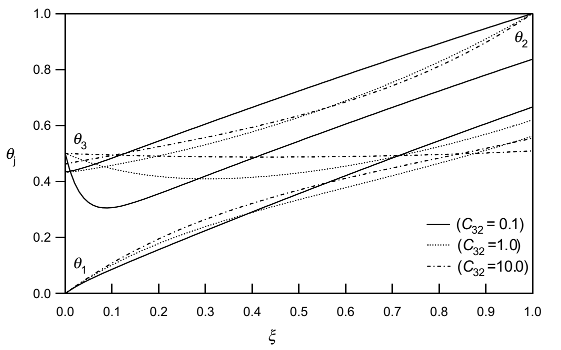

Fig. 7. Effect of C_{32} on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $\mathrm{NTU}_1 = 1.0$, and $\theta_{3,\mathrm{in}} = 0.5$.

streams 3 and 2. Thus, with the increase in the value of R_1, the difference between the temperature distributions of fluid streams 3 and 2 decreases and the temperature distribution of fluid 3 tends to follow that of fluid 2. Since the overall thermal resistance between fluids 3 and 1 is less than the overall thermal resistance between fluids 1 and 2 ($R_2 > 1$), the temperature distribution of fluid 1 follows that of fluid 3. The effect of R_2 on the fluid temperatures for case P2 is shown in Fig. 5. The thermal response of the fluid streams may be explained using arguments similar to those used for the R_1 effect.

The effect of C_{12} on the temperature distributions of all three streams is shown in Fig. 6 for case P2. C_{12} is defined as the ratio of the thermal capacity of fluid stream 1 to that for fluid 2. Therefore, as C_{12} increases while other design parameters are fixed, the thermal capacity of fluid 1 increases relative to the thermal capacity of fluid 2. In other words, if the specific heats of both fluids, $c_{p,1}$ and $c_{p,2}$, are assumed to be approximately equal, the mass flow rate of fluid 1 increases relative to fluid 2 with the increase in C_{12}. Thus the axial temperature rise in fluid stream 1 decreases with the increase in C_{12}. For very high values of C_{12} the temperature distribution of fluid 1 remains close to 0. This establishes very high temperature differences between fluid stream 1 and the other two streams near the entrance ($\xi = 0$) of the heat exchangers for case P1. For case P2 this results in high temperature gradients near $\xi = 0$ and $\xi = 1$. Therefore the other two fluid streams experience a rapid temperature drop near $\xi = 0$ for case P1 and near $\xi = 0$ and 1 for case P2. Since the overall thermal resistance to heat transfer between fluid streams 3 and 2 is less than that between fluid streams 2 and 1, the temperature distribution of fluid 2 follows the temperature distribution of fluid 3 more closely than fluid 1 ($R_1, R_2 > 1.0$). The effect of C_{32} on fluid temperatures for case P2 is shown in Fig. 7. The thermal response within the heat exchanger as a function of C_{32} may be explained using the arguments similar to those used for the C_{12} effect.

The effect of NTU_1 on the temperature distributions of all three streams is shown in Fig. 8 for case P2. NTU_1 is defined as the inverse of the product of the overall heat transfer resistance between fluid streams 1 and 2 and the thermal capacity of fluid 1. Therefore, an increase in the value of NTU_1 suggests a decrease in the value of either the thermal capacity of fluid 1 or the overall heat transfer resistance between fluid streams 1 and 2, or both.

As R_1 and R_2 are fixed, a reduction in thermal resistance between streams 1 and 2 will not cause any change in the temperature distributions of the different streams. However, if the increase in the value of NTU_1 is interpreted as the decrease in the thermal capacity of fluid 1, the temperature distributions of different streams shown in Fig. 7 can be easily explained. As NTU_1 increases, the mass flow rate of fluid 1 decreases, provided the specific heat of fluid 1 is assumed to be constant. Also, as C_{12}

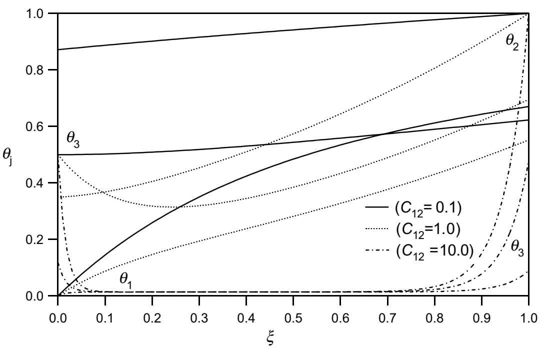

Fig. 6. Effect of C_{12} on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_1 = 2.0$, $R_2 = 1.5$, $C_{32} = 0.5$, $\mathrm{NTU}_1 = 1.0$, and $\theta_{3,\mathrm{in}} = 0.5$.

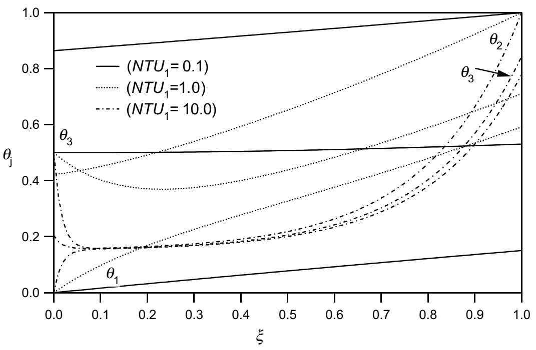

Fig. 8. Effect of NTU_1 on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, and $\theta_{3,in} = 0.5$.

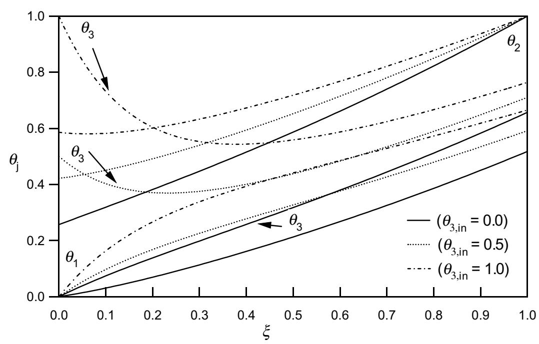

Fig. 9. Effect of $\theta_{3,in}$ on the temperature distribution of all the three streams along the length of the heat exchanger for case P2. Values of other parameters are: $R_1 = 2.0$, $R_2 = 1.5$, $C_{12} = 0.8$, $C_{32} = 0.5$, and $NTU_1 = 1.0$.

and C_{32} are constant, the mass flow rates of the other streams will also be reduced if their specific heats are also assumed to be constant. This results in steeper thermal gradients for all three streams as NTU_1 increases (Fig. 8). It is indicated in Fig. 8 that the increase in the value of NTU_1 increases the difference between the inlet and outlet temperatures for all three streams.

For case P2, as the value of NTU_1 is increased from 0.1 to 1, gradual changes in the temperature distributions of all the fluids occur. An increased temperature difference between the inlet and outlet temperatures of the three fluids is also observed. This behavior can be explained easily as before. As NTU_1 is increased from 1 to 10, a decrease in the mass flow rates of all the streams results in a very high temperature drop in fluid 2 as it enters the heat exchanger. As the resistance between fluids 2 and 3 is the minimum of the inter-fluid resistances, the temperature distribution of fluid 3 tries to follow the temperature distribution of fluid 2 and in the process fluid 2 loses considerable thermal energy at the heat exchanger entrance. This results in a slight increase in the temperature distribution of fluid 2 at the end of its flow. Since the overall heat transfer resistance between fluids 3 and 1 is more than the overall thermal resistance between fluids 3 and 2 but less than that between fluids 2 and 1, there will be more thermal interaction between fluids 1 and 3 than 1 and 2. This explains the behavior of the three streams for $NTU_1 = 10$ for case P2.

The effect of $\theta_{3,in}$ on the temperature distributions of all the three streams is shown in Fig. 9 for case P2. $\theta_{3,in}$ is defined as the ratio of the temperature differences between the inlet temperature of fluids 3 and 1 to fluids 2 and 1. Therefore, an increase in the value of $\theta_{3,in}$ results in more thermal energy transfer to fluid 1. The temperature differences between fluids 1 and 2 are subsequently reduced. Therefore an increase in $\theta_{3,in}$ results in a rise of the temperature distributions of the other two streams, fluids 1 and 2, (Fig. 9). Similarly, a decrease in the value of $\theta_{3,in}$ allows fluid 2 to cool down more. This reduces the thermal energy available to fluid stream 1 and therefore decreases the temperature distribution of fluid 1.

Several singularity cases can be identified from the solution which arise either due to the presence of multiple zero or non-zero eigenvalues (e.g., when $A = 0$ or when $B^2 - 4AC = 0$) [15–17] or due to one of the denominators in Eqs. (17)–(23) approaching zero for a given set of design parameters. These kinds of singularities are also present in all of the previously existing models. However, it should be noted that these singularity cases do not limit the real world usefulness of the developed solution or any of the previously existing models. As suggested by Luo et al. [3], if solutions are desired for singular cases, the values of the design parameters (e.g., C_{12} and C_{32}) should be perturbed to get approximate results. In all of the real world problems, solution of the required accuracy can be found by choosing values of the design parameters, very close to the exact ones. As mentioned before, the solution for the cases of multiple eigenvalues are presented in Appendix A for the sake of completeness.

4. Conclusions

A general, simple and easy to implement analytical model that enables heat exchanger design and analysis of three-fluid heat exchangers with three thermal communications for all flow arrangements is developed. The model is shown to reduce to three-fluid with two thermal communications and two-fluid heat exchanger models under proper conditions. Six non-dimensional design parameters are identified and their effect on the temperature distributions of the different fluid streams is presented and discussed. The model shows that the presence of three coupled thermal interactions makes temperature distributions of the different streams difficult to assess due to slight changes in the design parameters. The new model is useful in the understanding of the effect of single and combined parameters on the temperature distribution for a stream.

Appendix A

As mentioned previously, Eqs. (9), (10) and (12) along with the non-dimensional boundary conditions given in Table 1 for all four fluid flow arrangements represent a complete eigenvalue problem. This eigenvalue problem can be solved to obtain three eigenvalues D_1, D_2, and D_3 which will determine the temperature distribution of all three fluids for all four cases. It may happen for certain combinations of design parameters that two of these three possible eigenvalues are equal (e.g., when $A = 0$ or when $B^2 - 4AC = 0$). In these cases of multiple eigenvalues, the solution in an abridged form can be presented as follows:

$$\theta_i = a_{i,1}\exp(D_1\xi) + (a_{i,2} + a_{i,3}\xi)\exp(D_3\xi) \quad i = 1,2,3 \tag{A.1}$$

It has been assumed here that $D_2 = D_3$ and D_1 is the eigenvalue which is different from the other two (note that D_1 is not assumed to always equal to zero as before). Therefore, it should be noted here that D_2 and D_3 could be zero as well as non-zero eigenvalues. The constants a_{ij} can be determined as follows:

$$s_j = a_{1j}/a_{2j} \quad j = 1,2,3$$
$$u_j = a_{3j}/a_{2j} \quad j = 1,2,3 \tag{A.2}$$

with,

$$s_j = \frac{R_1 + R_2 + R_1R_2 + \frac{i_2D_jR_2}{\mathrm{NTU}_1C_{12}}}{R_1 + R_2 + R_1R_2 + \frac{i_1D_jR_1}{\mathrm{NTU}_1}} \quad j = 1,3 \tag{A.3}$$

$$s_2 = \frac{Y_3Z_2 - Y_2Z_3}{Y_1Z_2 - Y_2Z_1} \tag{A.4}$$

$$u_j = \frac{\left[\frac{i_1D_j}{\mathrm{NTU}_1} + 1 + R_2\right]s_j - 1}{R_2} \quad \text{or}$$
$$u_j = \frac{\left[\frac{i_2D_j}{\mathrm{NTU}_1C_{12}} + 1 + R_1\right] - s_j}{R_1} \quad j = 1,3 \tag{A.5}$$

$$u_2 = \frac{Y_1Z_3 - Y_3Z_1}{Y_1Z_2 - Y_2Z_1} \tag{A.6}$$

with,

$$Z_1 = i_3/i_2u_3C_{32} - R_1 \tag{A.7}$$

$$Z_2 = i_3D_3/(\mathrm{NTU}_1C_{12}/C_{32}) + R_1 + R_2 + R_1i_3/i_2u_3C_{32} \tag{A.8}$$

$$Z_3 = R_1 + i_3/i_2u_3C_{32}(1 + R_1 + i_2D_3/(\mathrm{NTU}_1C_{12})) \tag{A.9}$$

$$Y_1 = i_1/i_2s_1C_{12} + i_1D_3/\mathrm{NTU}_1 + 1 + R_2 \tag{A.10}$$

$$Y_2 = i_1/i_2s_1C_{12}R_1 - R_2 \tag{A.11}$$

$$Y_3 = 1 + i_1/i_2s_1C_{12}(1 + R_1 + i_2D_3/(\mathrm{NTU}_1C_{12})) \tag{A.12}$$

and,

$$\begin{bmatrix} s_1 & s_2 & 0 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \begin{bmatrix} a_{21} \\ a_{22} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \theta_{3,\mathrm{in}} \end{bmatrix} \tag{A.13}$$

where, for cases

$$\text{P2 and P3,} \quad v_j = \exp(D_j) \quad j = 1,2,3 \tag{A.14}$$

$$\text{P1 and P4,} \quad v_j = 1 \text{ and } v_3 = 0 \quad j = 1,2 \tag{A.15}$$

$$\text{P1 and P2,} \quad w_j = u_j \text{ and } w_3 = 0 \quad j = 1,2 \tag{A.16}$$

$$\text{P3 and P4,} \quad w_j = u_j\exp(D_j) \quad j = 1,2,3 \tag{A.17}$$

The solution presented in Eqs. (A.1)–(A.17) for multiple eigenvalues has little importance for real world problems and is presented only for the sake of completeness. As mentioned previously, the general solution presented in Eqs. (13)–(27) can still be used in all of the singularity cases by introducing small perturbations in the values of the design parameters (e.g., C_{12} and C_{32}) to get approximate results.

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